Math 1653
Analysis I

Tuesday 3 October 2023

Problem session by Adam Abrams

Topics

Limits

- Sequences
- Functions
- Continuity

Derivative calculations

- Power Rule
- Trig, log, exp
- Product Rule
- Chain Rule

Derivative applications

- Tangent lines
- Increasing and decreasing
- Concavity
- Min and max

Integrals

- Indefinite
- Definite
- Applications

Some students may already know some of these topics, but we will cover them all during this semester.

Course format

Lecture (Wykład)

- Wednesdays 11:15 - 13:00 with dr Adam Abrams.

Problem session (Ćwiczenia)

- Tuesdays 18:55 - 20:35 with dr Adam Abrams,
- Thursdays 7:30 - 09:00 with dr Artur Rutkowski.

Lecture slides, tasks lists, and course policies are available at

theadamabrams.com/1653

Grading policy

The same grade is used for 1653W and 1653C.

- Six quizzes (5 points each), but the lowest score is ignored!
- Two exams (15 points each).
- Participation (5 points).

This makes 5×5 + 15+15 + 5 = **60** total possible points.

Points	[0, 30)	[30, 36)	[36, 42)	[42, 48)	[48, 54)	[54, 60]
Grade	2.0	3.0	3.5	4.0	4.5	5.0

Grading policy

The same grade is used for 1653W and 1653C.

Points	[0, 30)	[30, 36)	[36, 42)	[42, 48)	[48, 54)	[54, 60]
Grade	2.0	3.0	3.5	4.0	4.5	5.0

More than 4 unexcused absences after 6 Oct → course grade 2.0.

You can work together on task lists (which are not graded), but quizzes and exams are individual. All work can be checked in one-on-one meeting with either instructor.

- Cheating on a quiz → quiz grade 0.
- Cheating on exams → course grade 2.0.

Accessibility

Department of Accessibility and Support for People with Disabilities (DDO)

- Office: C-13 rooms 109 and 107
- Telephone: 71 320 43 20
- Website: https://ddo.pwr.edu.pl/
- Email: pomoc.n@pwr.edu.pl

If you need any kind of accommodation, please write me an email.
I am happy to help.

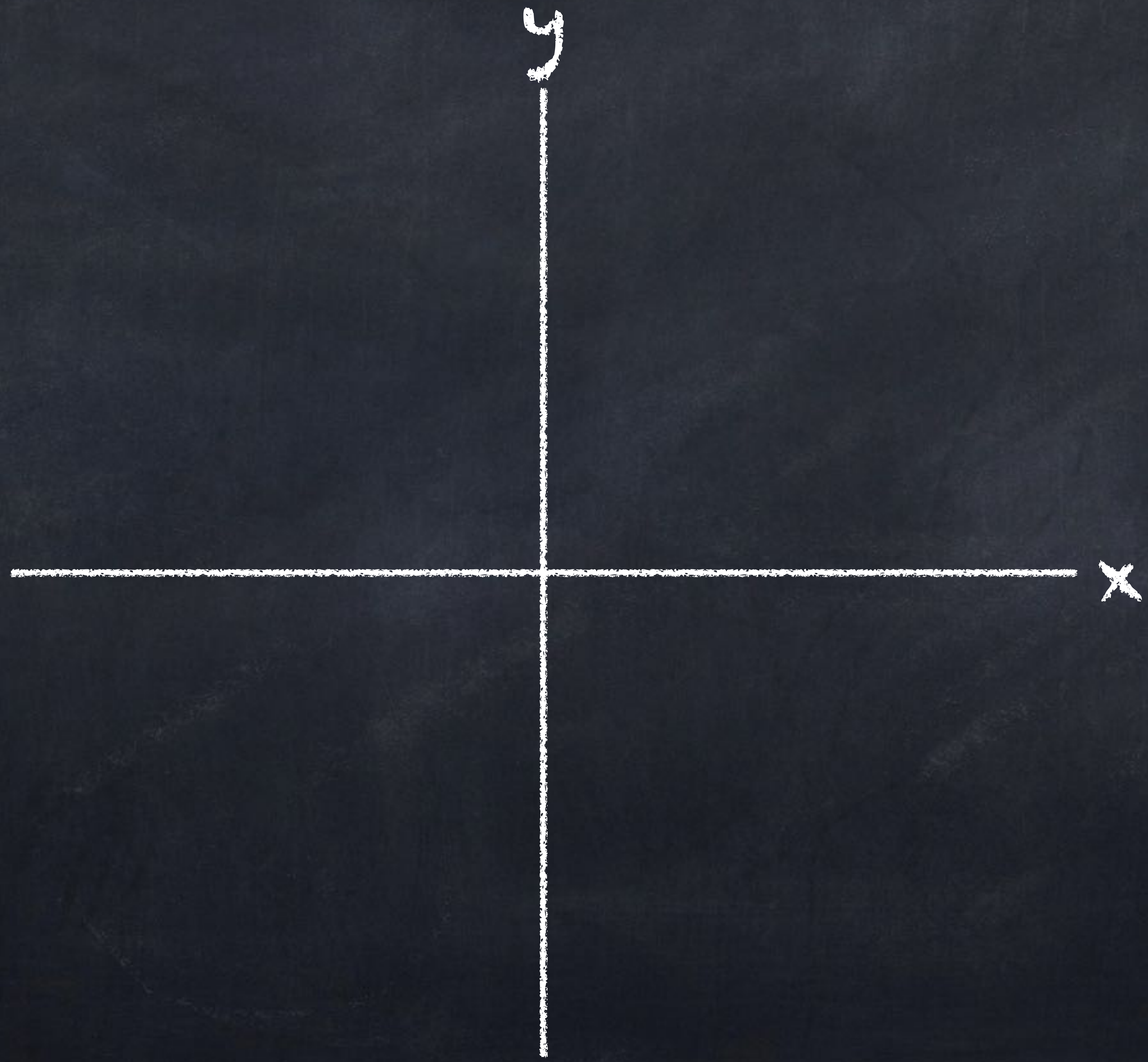
English language and some polls

poles

Poles

polls

Draw $y = \frac{1}{2}x + 3.$



$y = x^2$

$y = x^2 - 1$

$y = (x - 1)^2$

Task: Solve $2x^2 + 7x - 15 = 0.$

Task: Give an equation for the line through the point $(5, 1)$ with slope 3.

Algebra review maybe

- "$6 \times a$" and "$6 \cdot a$" and "$6a$" all mean six times a.
- $6(a+b)$ can be re-written as $6a+6b$.
- $3x-12$ can be re-written as $3(x-4)$. This is "factoring".
- $(x+7)(y+2)$ can be expanded to $xy+2x+7y+14$.
- $(x+7)^2$ can be expanded to $x^2+14x+49$.

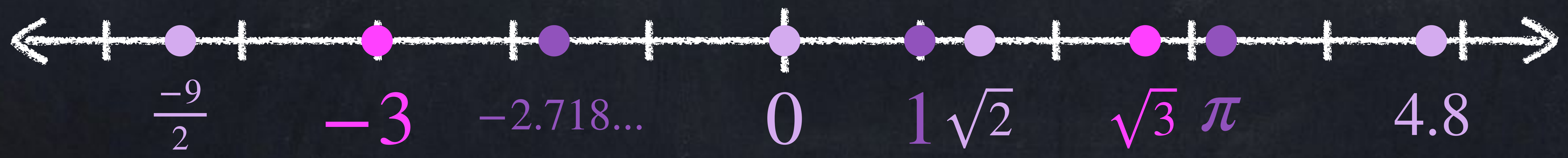
  In general, $(a+b)^2$ expands to $a^2+2ab+b^2$.
- $x^2+14x+49$ can be factored as $(x+7)^2$.

Be careful!

- $(a \times b)^2$ can be re-written as $a^2 \times b^2$.
- $(a + b)^2$ can **not** be re-written as $a^2 + b^2$.
  - Try it with actual numbers:
    $(2+3)^2 = 5^2 = 25$, but $2^2 + 3^2 = 4 + 9 = 13$.
  - Testing specific numbers can only show you when a rule is *false*. It cannot guarantee that a rule is correct because you might pick numbers where it accidentally works, like $(0+0)^2 = 0 = 0^2 + 0^2$.
- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$
- $\sin(a \cdot b) \neq \sin(a) \cdot \sin(b)$

Types of numbers

- **Natural** numbers: $0, 1, 2, 3, 4, \ldots$
  - In some books, only $1, 2, 3, 4, \ldots$
- **Integers**: $\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$
- **Rational** numbers are all the numbers that *can* be written as one integer divided by another. Examples: $\frac{1}{2}, \frac{-2}{3}, 1.5, 8, 0, \frac{-5}{4}$
- **Real** numbers are all the values on a number line. Examples:



Types of functions

- **Polynomial**:

  😉$x^n + \cdots +$😊$x^2 +$🤨$x +$😃

  Like with rational numbers, a function can be a polynomial even if it is written in a different way. Example: $(x+4)^2$.

- **Exponential function**:

  😐 $\cdot$ 🙂^{x}

- **Trig function**:

  😐$\sin($🤨$x +$ 😕$) +$ 😃

  and similar for $\cos$, $\tan$, $\cot$, $\sec$, $\csc$.

- **Absolute value**

Absolute value

- Algebra idea: make numbers positive
- Geometry idea: measure distance

- We write x for the **absolute value** of x.

- Examples: 5 is 5 $\quad -3$ is 3 $\quad \left|-\frac{9}{2}\right|$ is $\frac{9}{2}$ $\quad 37.2$ is 37.2

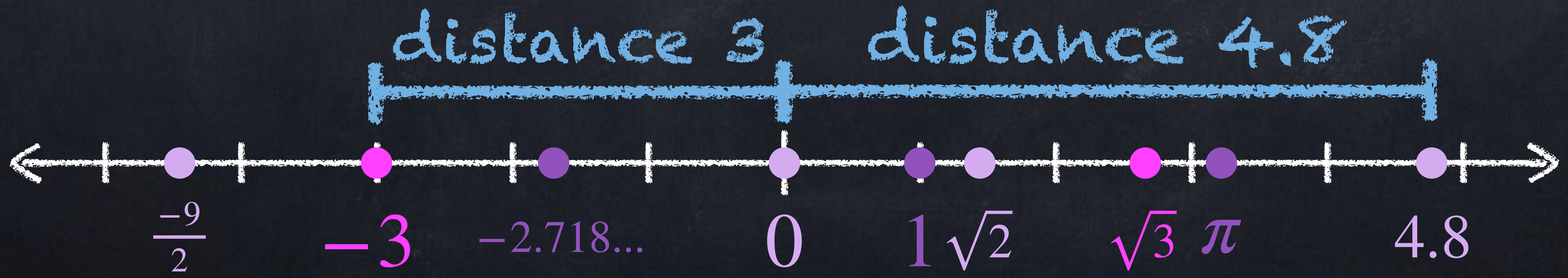
- Definition, version 1: $$x = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

Absolute value

- Algebra idea: make numbers positive
- Geometry idea: measure distance

- We write x for the **absolute value** of x.

- Definition, version 2: x is the distance between 0 and x.



Absolute value

What does the graph $y = \quad x \quad$ look like?

Equations of Lines

Slope-intercept form:

$$y = mx + b.$$

- This line has slope m.
- This line includes the point $(0, b)$.

https://www.desmos.com/calculator/ae5cdos3mv

Point-slope form:

$$y - b = m(x - a) \qquad \text{or} \qquad y = b + m(x - a)$$

- This line has slope m.
- This line includes the point (a, b).

https://www.desmos.com/calculator/t2iug79i3m

Expanding

$(x+7)^2 = x^2 + 14x + 49$

$(x+2)^3$ = ?

The Binomial Theorem gives rules for higher powers:

$$(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + C(n,3)x^{n-3}y^3 + \cdots + C(n,n-1)xy^{n-1} + C(n,n)y^n$$

where the coefficients

$$C(n,k) = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 2\cdot 1}$$

can also be found from Pascal's Triangle.

Expanding

$(x+7)^2 = x^2 + 14x + 49$

$(x+2)^3$ = ?

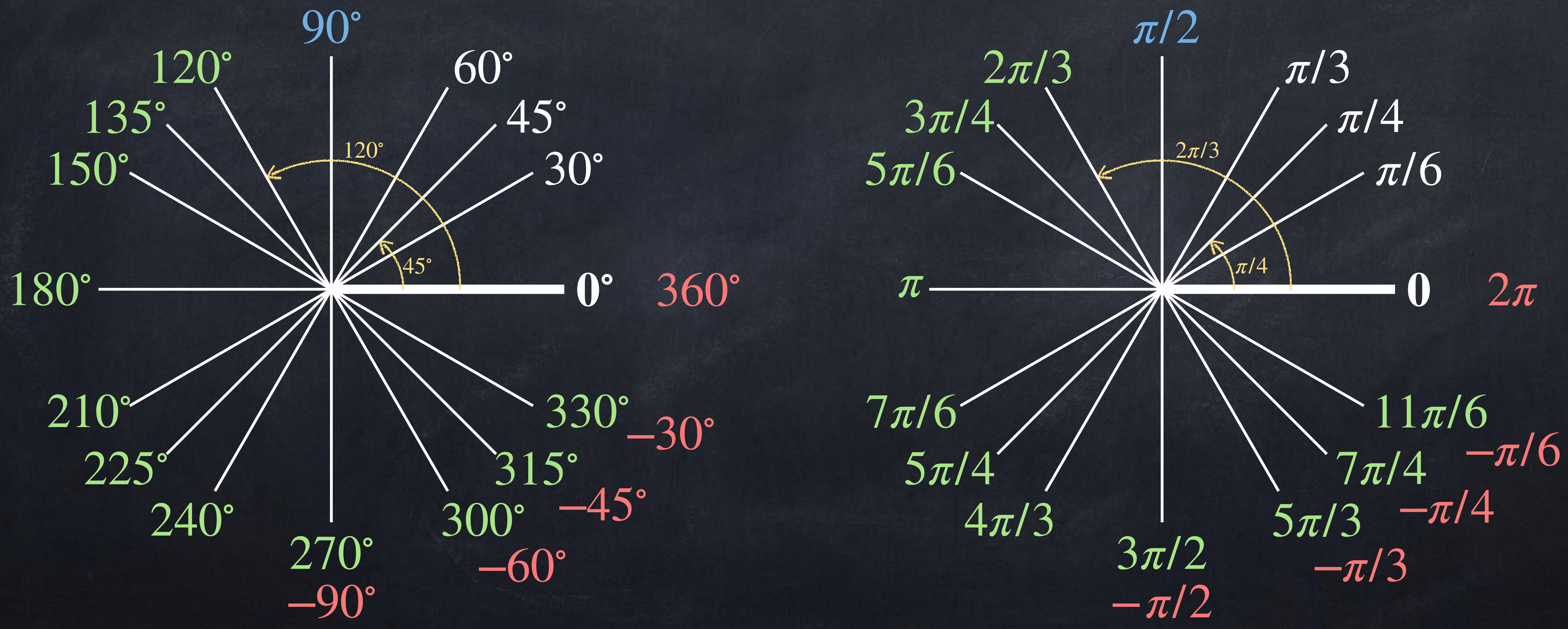
The Binomial Theorem gives rules for higher powers:

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots + nxy^{n-1} + y^n.$$

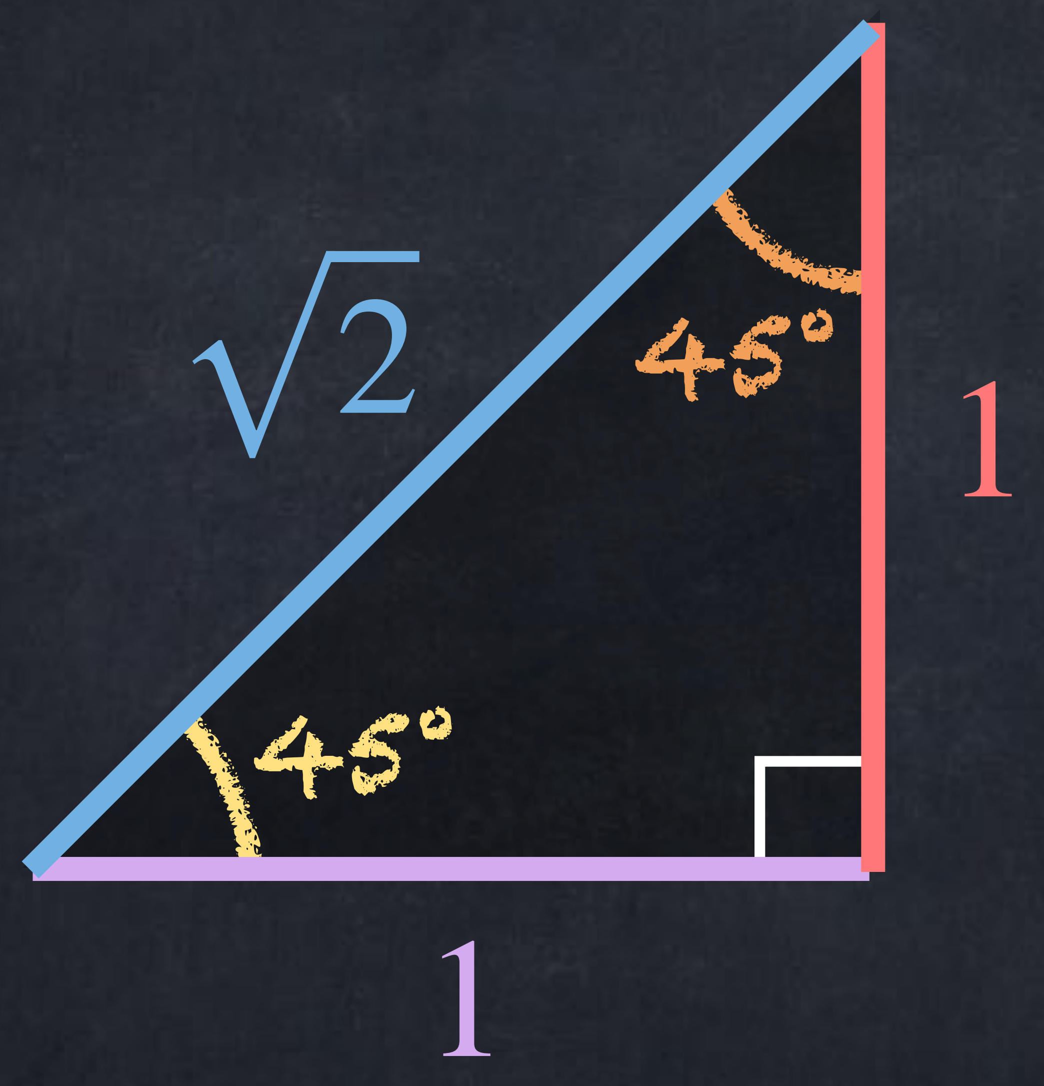
The expansions for $(a+b)^2$ and $(a+b)^3$ are the most important to know. Higher powers are used less frequently.

Measuring angles

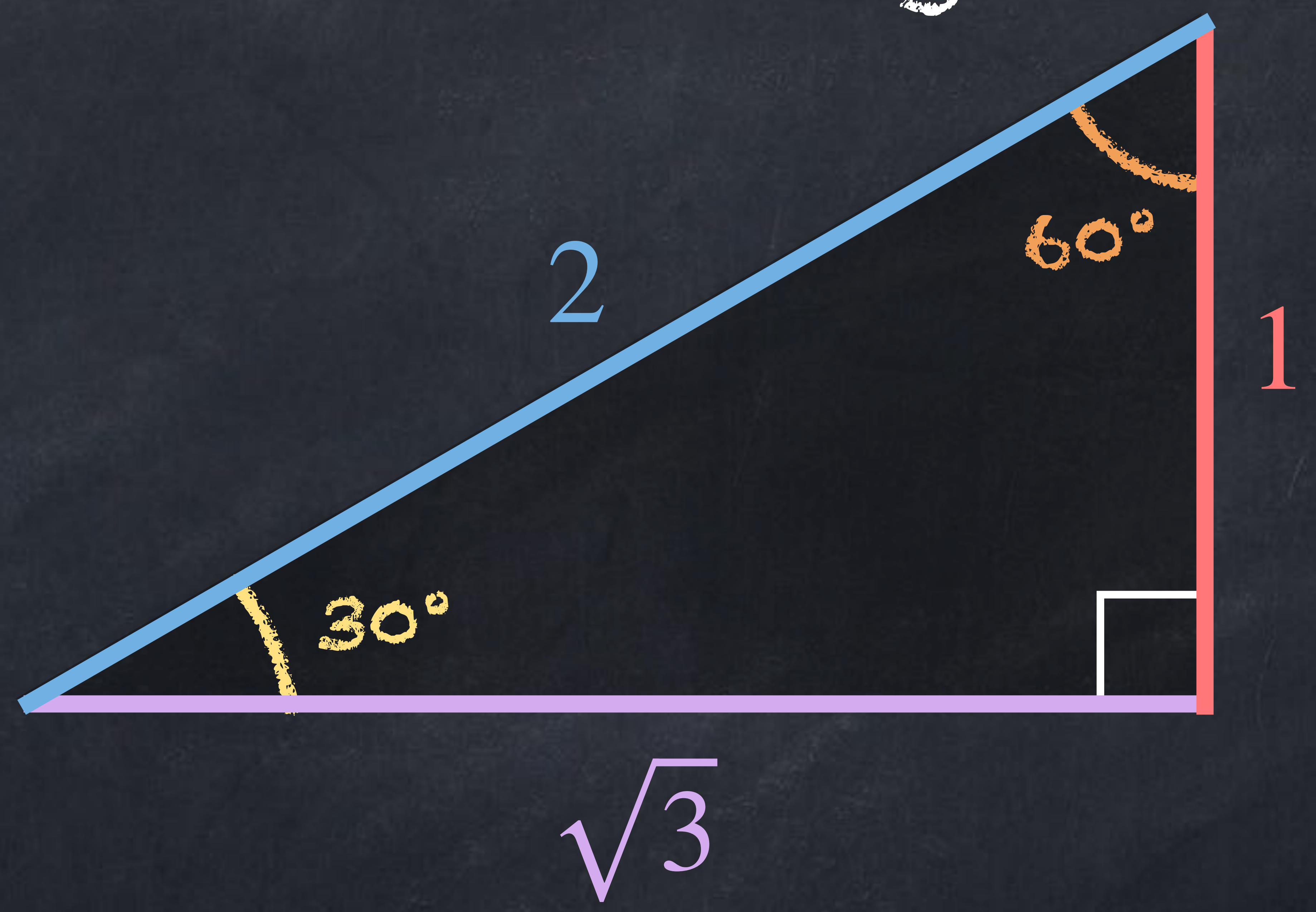
In this class we will mostly use radians (after we learn about derivatives, I can explain why).



"45-45-90 triangle"

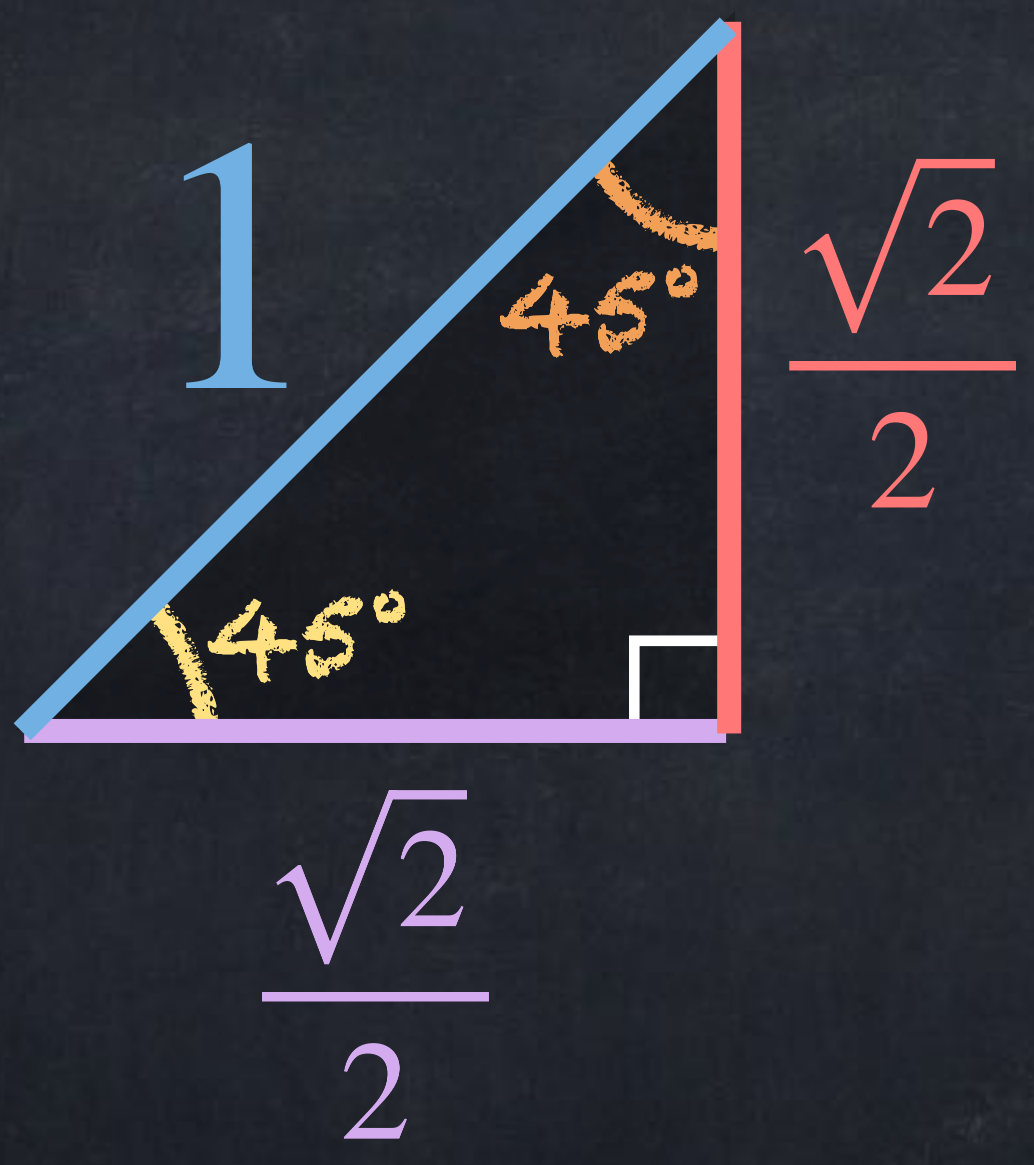


"30-60-90 triangle"

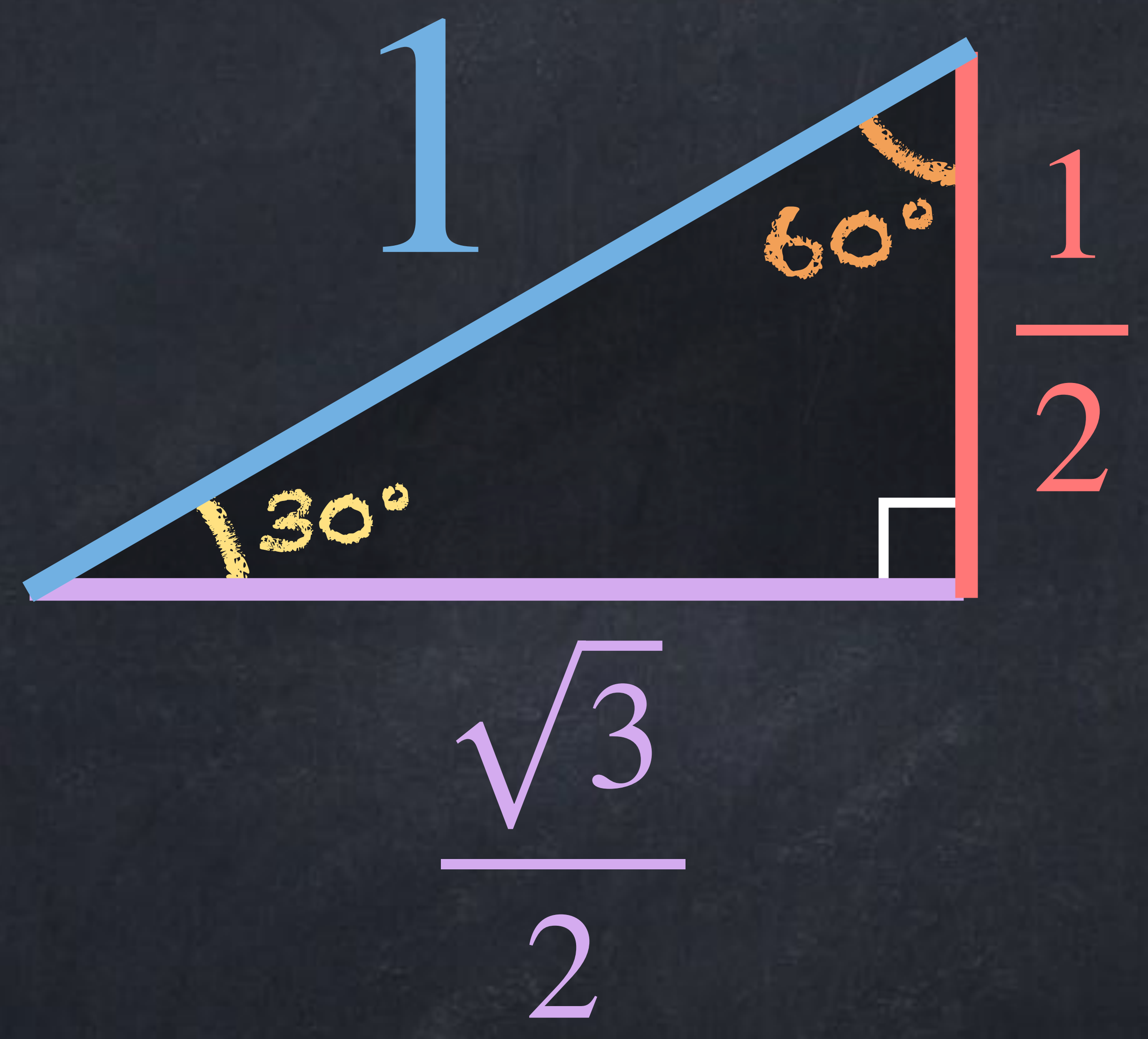


It's possible to define $\sin$ and $\cos$ values by dividing one side length (e.g., $\sqrt{3}$) by another (e.g., 2), but for this class it is better to re-scale the triangles to have a hypotenuse of exactly 1.

"45-45-90 triangle"



"30-60-90 triangle"

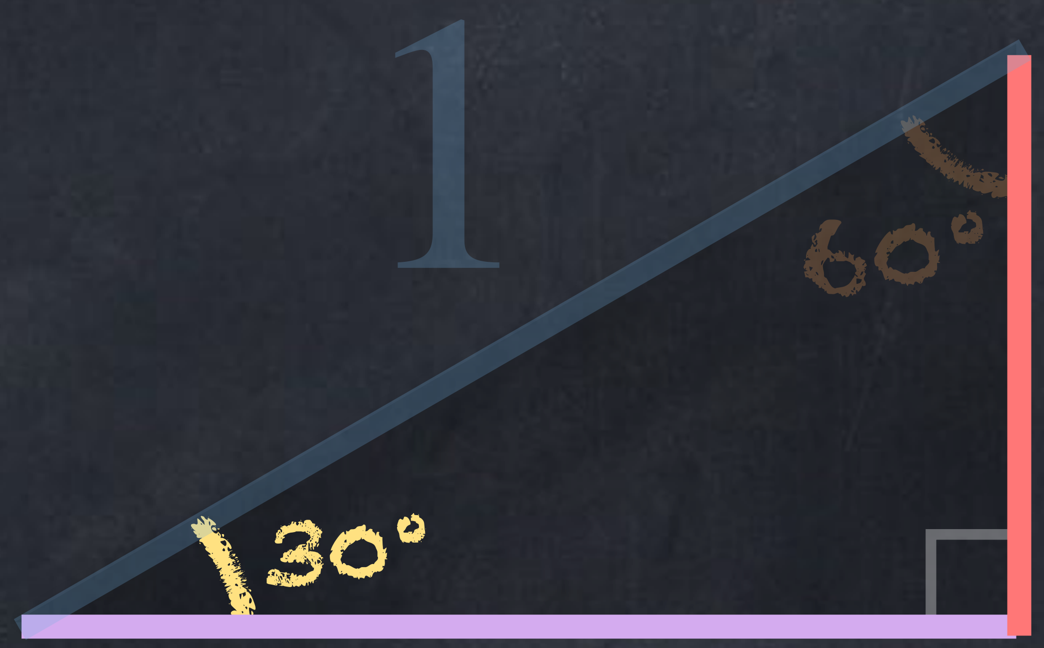


Memorize these!

Trig functions

$$\frac{\sqrt{2}}{2} = \sin(45°)$$

$$\frac{\sqrt{2}}{2} = \cos(45°)$$



$$\frac{1}{2} = \sin(30°)$$

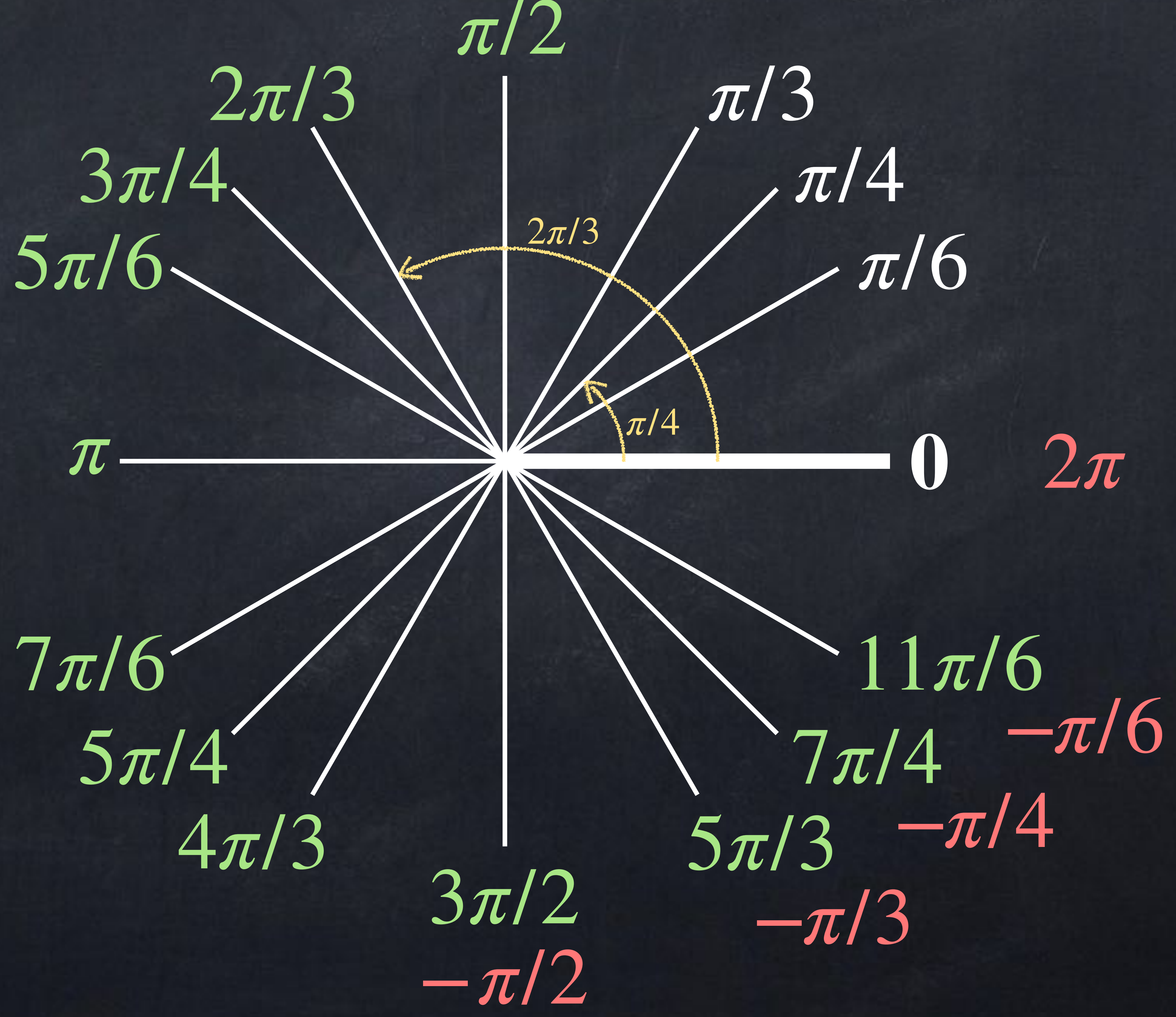
$$\frac{\sqrt{3}}{2} = \cos(30°)$$

Memorize these!

Trig functions

$\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$

$\left(\cos\frac{3\pi}{2}, \sin\frac{3\pi}{2}\right)$ is $(0, -1)$



Exponential functions

Graph $y = 2^x$.

Graph $y = 3^x$.

Graph $y = 2.71^x$.

The number e is approximately 2.71828.
So the graph of e^x looks like 2.71^x.

Graph $y = 2^{-x}$.

Opposites of exponents

- If $x^3 = 27$ then $x = \ldots$
- If $x^2 = 64$ and $x > 0$ then $x = \ldots$
- If $x^2 = 5$ and $x > 0$ then $x = \ldots$
- If $2^x = 8$ then $x = \ldots$
- If $2^x = 9$ then $x = \ldots$

Definition: if $x^2 = a$, then a is exactly $\sqrt{x}$.

Definition: if $2^x = a$, then a is exactly $\log_2(x)$.

Definition: if $e^x = a$, then a is exactly $\ln(x)$.